Optimization of Passenger Railway System Design

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System design considers trade-off between cost and reliability

- A rail system consists of a number of subsystems, and each has its own cost and reliability
- Planners have to carefully allocate the reliability and budget by examining the trade-off between cost and reliability
Life Cycle of Railway System (V representation in EN50126)

1. Concept
2. System definition
3. Risk analysis
4. System requirements
5. Assignment
6. Construction
7. Fabrication
8. Installation
9. System validation
10. Approval; acceptance
11. Operation; maintenance
12. Capture performance
13. Change; retrofitting
14. Suspension; disposal

Apportionment of system requirements

Top-down
System reliability cannot reflect its impact to passengers

- Various reliability allocation methods have been developed in the past.
  - Weighting Method
  - Optimization Method

- For rail system, reliability, so called system reliability, is defined as the mean time between failures (MTBF) or mean distance between failures (MDBF)

However, this attribute does not consider its effect on passengers (the consequence of failure)
System Reliability vs. Service Reliability

- Failure frequency ➔ MTBF ➔ System Reliability
- Consequence of failure ➔ Delay ➔ Service Reliability

Both of them have same system reliability, but their effects on passengers are quite different.

Minor Train failure
MTBF: 100,000 train-hour
1 minute delay (for one train)

Communication failure
MTBF: 100,000 train-hour
1 hour delay (for multiple trains)
Service reliability (e.g. delay or on-time percentage) is more favorable than system reliability because it considers customers’ satisfaction.

Service reliability can be obtained by the relationship between service reliability and system reliability.

$$\text{Estimated Delay}_{ik} = \left( \frac{\text{Total Operational Time}}{\text{MTBF}_{ik}} \right) \times (\text{Average Delay})_{ik}$$
LCC for railway systems typically includes capital investment, operating cost, and maintenance cost within the planning period.

$LCC = \text{Capital Cost} + \text{Operating Cost} + \text{Maintenance Cost}$

Employing LCC is more appropriate in decision making than solely employing capital investment:

- Some products have low capital investment but high operating and maintenance costs (e.g. ballast track)
- Others have high capital investment but low operating and maintenance costs (e.g. slab track)
Key Elements in Rail System Design – System Reliability

- System reliability is defined as MTBF or MDBF, and Failure Rate (= 1/MTBF)
- The **higher** MTBF or MDBF results in **higher** system reliability
- Information about **system reliability** and LCC can be obtained from suppliers

![Diagram of rail system design showing key elements]

- **Time or distance between failures**
  - Failure
  - Failure
  - Failure
  - Failure
  - Exposure

Service Reliability

Rail System Design

Life Cycle Cost

System Reliability
Key Elements in Rail System Design – Service Reliability

- Service reliability identifies the effect on passengers
- Target Service Reliability – On-time arrival percentage (with no buffer): proportion of on-time operations in terms of total system operating time (in train-hour)

\[ r_{ser} = \left( \frac{P - \sum_{j \in J} D_j f_j^B}{P} \right) \times 100\% \]

- On-time arrival percentage (service reliability)
- Total Delay time (train-hour)
- On-time arrival time (train-hour)
- Total system operational time (train-hour)

- \( r_{ser} \) = On-time arrival percentage
- \( P \) = Total system operational time (in train-hour) in a defined period
- \( D_j \) = Delay (in train-hour) of subsystem j within operational time or distance
- \( f_j^B \) = Failure rate
Optimization Framework with MCM and MRM (1st Development)
Alternative Evaluator (AE)

- AE evaluates all possible alternatives and generates an investment alternative table with their LCC, system reliability, and service reliability.
- Service reliability needs to be computed based on system reliability.

Number of failures

\[ D_{ik} = \left( \frac{T_i}{M_{ik}} \right) NQ_{ik} \]

\[ \forall i \in I, k \in K \]

- \( D_{ik} \) = Delay (in train-hour) of subsystem \( i \) with alternative \( k \)
- \( M_{ik} \) = MTBF or MDBF of subsystem \( i \) with alternative \( k \)
- \( T_i \) = Operational time or distance of subsystem \( i \) in a defined period
- \( N \) = Average number of online trains
- \( Q_{ik} \) = Average delay (in hours) from a failure of subsystem \( i \) with alternative \( k \)

- \( Q_{ik} \) is estimated using historical data from similar systems or determined by simulations based on the service effect from possible types of failures.
## Alternative Evaluator (AE)

### Investment Alternatives

- **Subsystem Alternatives**
  
<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Alternatives</th>
<th>MDBF ($M_{ik}$)</th>
<th>LCC ($C_{ik}$)</th>
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<td></td>
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</tbody>
</table>

- **Alternative Evaluator (AE)**

- **Investment Alternatives Table**

- **Demand**

![Diagram showing the relationship between Investment Alternatives, Alternative Evaluator, Investment Alternatives Table, and Demand.](image)
Investment Selector (IS)

- IS identifies the best alternative for every subsystem according to acceptable LCC or service reliability
  - Minimize Cost Model (MCM): Minimizing total LCC according to acceptable service reliability
  - Maximize Reliability Model (MRM): Maximizing service reliability according to available LCC
Minimize Cost Model

\[ \text{Min} \quad \sum_{i \in I} \sum_{k \in K} C_{ik} \delta_{ik} \quad \rightarrow \quad \text{Minimize total LCC} \]

**S.t.**

\[ \sum_{k \in K} \delta_{ik} = 1 \quad \forall i \in I \quad \rightarrow \quad \text{Only one alternative can be chosen for a subsystem} \]

\[ d_i = \sum_{k \in K} D_{ik} \delta_{ik} \quad \forall i \in I \quad \rightarrow \quad \text{Compute delay for each subsystem} \]

\[ \left( \frac{P - \sum_{i \in I} d_i}{P} \right) \times 100\% \geq R \quad \rightarrow \quad \text{Fulfill the service reliability requirement} \]

\[ \delta_{ik} \in \{0,1\} \quad \forall i \in I, k \in K \]

\[ d_i \geq 0 \quad \forall i \in I \]

---

**Decision Variables**

<table>
<thead>
<tr>
<th>( \delta_{ik} )</th>
<th>whether alternative is selected</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_i )</td>
<td>the delay of subsystem ( i )</td>
</tr>
</tbody>
</table>

**Parameters**

| \( C_{ik} \) | LCC of alternative |
| \( D_{ik} \) | delay of alternative |
| \( P \) | total operational time per year |
| \( R \) | design service reliability |
Maximize Reliability Model

\[
\text{Max} \left( P - \sum_{i \in I} d_i \right) \times 100\%
\]

\[\text{Maximize service reliability}\]

\[\sum_{k \in K} \delta_{ik} = 1 \quad \forall i \in I\]

\[\text{Only one alternative can be chosen for a subsystem}\]

\[d_i = \sum_{k \in K} D_{ik} \delta_{ik} \quad \forall i \in I\]

\[\text{Compute delay for each subsystem}\]

\[\sum_{i \in I} \sum_{k \in K} C_{ik} \delta_{ik} \leq B\]

\[\text{Constraint on LCC}\]

and

\[\delta_{ik} \in \{0, 1\} \quad \forall i \in I, k \in K\]

\[d_i \geq 0 \quad \forall i \in I\]

<table>
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Two case studies to demonstrate the potential use

- Two case studies with empirical data obtained from a rail system in Taiwan were performed to show the potential use of the proposed method

  - **Case I : New System Design**
    - Designing a new passenger rail system
    - Selecting appropriate alternatives for subsystems according to **design service reliability → MCM**

  - **Case II : Existing System Improvements**
    - Improving the reliability of an existing rail system
    - Subject to constraint on **available increment in LCC → MRM**
Case I: New System Design

- 25-km passenger rail system
- Estimated demand is 140,000 passengers per day
- Six subsystems: train, signal, communication, electricity, station, and track
- Design service reliability (on-time arrival percentage) is from 95% to 99%, with 1% increments

### Investment Alternatives

![Graph showing investment alternatives](image)

<table>
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<tr>
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High design service reliability results in high MDBF and LCC

- MCM efficiently solved this problem by using CPLEX within seconds

<table>
<thead>
<tr>
<th>Subsystem</th>
<th>Design Service Reliability</th>
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<tbody>
<tr>
<td></td>
<td>95%</td>
</tr>
<tr>
<td><strong>MDBF (train-km)</strong></td>
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<tr>
<td><strong>Total</strong></td>
<td>92.83</td>
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</table>
The difference in MDBF among subsystems becomes obvious as service reliability level increase.
Increase in total LCC from 95% to 97% is modest but very sharp from 97% to 99% because of the nonlinear relationship between cost and reliability.
Case II: Existing System Improvement

- Demand is the same as Case I and service reliability of the existing system is 97% with improvement LCC from 1 ~ 5 billions.
- Not all of the subsystems can be easily changed so we consider alternatives for communication, electricity, and track in this case.
- MRM efficiently solved this problem by using CPLEX within seconds.

### Subsystem Alternatives

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Resulting MDBF from MRM

- Track has the most significant increase in MDBF, followed by communication and electricity.
More LCC have been allocated to electricity, and communication for all scenarios.

Impact on delay from communication and electricity failures is more severe than that from track failures.
An integrated optimization framework for rail system design is developed by minimizing LCC and service unreliability (delay cost)
The objective function considers the trade-off between cost and reliability, by minimizing total LCC and delay cost.

Min
\[
\sum_{i \in I} \sum_{n \in N} C_{in} \delta_{in} + HT \sum_{j \in J} D_j f^B_j
\]

- \( \delta_{in} \): a binary variable denoting the selection of the alternative
- \( f^B_j \): a continuous variable denoting the failure rate of subsystem \( j \)
Types of Constraints

**Group 1 – Alternatives Selection**

\[
\sum_{n \in N_i} \delta_{in} = 1 \quad \forall i \in I, N_i \in N
\]

\[
\sum_{k \in K} y_{jk} = 1 \quad \forall j \in J_r
\]

**Group 2 – Failure Rate Computation**

\[
f_{jk}^C \geq F_{in}^C + m(1 - y_{jk}) \quad \forall (i,n) \in V_k, k \in K_j, j \in J_s
\]

\[
f_{jk}^C \geq \prod_{(i,n) \in V_k} F_{in}^C + m(1 - y_{jk}) \quad \forall k \in K_j, j \in J_p
\]

**Group 3 – System Failure Rate**

\[
f_j^B \geq f_{jk}^C \quad \forall k \in K_j, j \in J_r
\]

\[
f \geq f_j^B \quad \forall j \in J_r
\]

**Group 4 – System Requirements**

\[
\sum_{k \in K} f_{jk}^m y_{jk} \leq G_j \quad \forall j \in J_r
\]

\[
f \leq E
\]

\[
\sum \sum C_{in} \delta_{in} \leq B
\]

\[
\left( P - \sum_{j \in J_r} D_j f_j^B \right) \times 100\% \geq R
\]
Group 1 – Alternatives Selection

\[
\sum_{i \in I, N_i \in N} \delta_{in} = 1 \quad \forall i \in I, N_i \in N
\]

→ **Component Selection**

\[
\sum_{k \in K} y_{jk} = 1 \quad \forall j \in J_r
\]

→ **Combination Selection**

\[
\sum_{(i,n) \in V_k} \delta_{in} \leq y_{jk} + (U_j - 1) \quad \forall k \in K_j, j \in J_r
\]

→ **Relation between Combination and Component Selection**

\[\delta_{in} \text{ a binary variable denoting the selection of the alternative}\]

\[y_{jk} \text{ a binary variable denoting the selection of the combination}\]
Group 2 – Failure Rate Computation

\[ f^C_{jk} \geq F^C_{in} \cdot m(1 - y_{jk}) \quad \forall (i, n) \in V_k, k \in K_j, j \in J_s \]

→ Failure Computation for Components with “OR” Relationship

\[ f^C_{jk} \geq \prod_{(i,n) \in V_k} F^C_{in} \cdot m(1 - y_{jk}) \quad \forall k \in K_j, j \in J_p \]

→ Failure Computation for Components with “AND” Relationship

\[ y_{jk} \quad a \ binary \ variable \ denoting \ the \ selection \ of \ the \ combination \]
\[ f^C_{jk} \quad a \ continuous \ variable \ denoting \ the \ failure \ rate \ of \ each \ combination \ k \ in \ subsystem \ j \]
Group 3 – System Failure Rate

\[ f_j^B \geq f_{jk}^C \quad \forall k \in K_j, j \in J_r \]

Returning the Subsystem Failure Rate

\[ f \geq f_j^B \quad \forall j \in J_r \]

Returning the System Failure Rate

\[ f_{jk}^C \quad \text{a continuous variable denoting the failure rate of each combination } k \text{ in subsystem } j \]

\[ f_j^B \quad \text{a continuous variable denoting the failure rate of subsystem } j \]

\[ f \quad \text{a continuous variable denoting the failure rate of the system} \]
Group 4 – System Requirements

\[ f \leq E \]

\[ \sum_{k \in K} f_{jk} y_{jk} \leq G_j \quad \forall j \in J_r \]

\[ \sum_{i \in I} \sum_{n \in N} C_{in} \delta_{in} \leq B \]

\[ \left( \frac{P - \sum_{j \in J_r} D_j f_j^B}{P} \right) \times 100\% \geq R \]

→ Constraint on total system reliability
→ Constraint on system reliability for each subsystem
→ Constraint on LCC budget
→ Constraint on service reliability

**IM-B** : IM + Constraint of LCC

**IM-R** : IM + Constraint of service reliability

**IM-BR** : IM + Constraints of LCC and service reliability
25-km passenger rail system
Estimated demand is 140,000 passengers per day

System structure:
- Five subsystems
- Some with sub-systems
Alternatives with Specific Cost and System Reliability Information
## Investment Alternatives Table

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<tr>
<th>Subsystem</th>
<th>Component</th>
<th>Alternatives</th>
<th>Failure Rate</th>
<th>LCC($C_{in}$) (billion dollars)</th>
<th>Delay ($D_j$) (train-hours)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rolling Stock</td>
<td>Rolling</td>
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Optimal Investment Plan

- IM efficiently solved this problem by using CPLEX within seconds.
- The resulting LCC is $97.25 billion NTD, the total delay cost is $7.23 billion NTD, and service reliability in on-time arrival rate is 97.84%.
- This allocation demonstrates the optimal balance between LCC and service reliability at a given design time value ($T = 64,742$ NTD/train-hour).

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<th>Subsystem</th>
<th>Failure rate</th>
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Pareto Front of Optimal Allocation

- Each point is the optimal balance between LCC and service reliability at a specific time value.

A higher time value \((T)\) increases the relative importance of service reliability and causes the results to move towards to the left, indicating higher service reliability (lower delay cost).

\[
\text{Min } \sum_{i \in I} \sum_{n \in N} C_{in} \delta_{in} + HT \sum_{j \in J_r} D_j f_j^B
\]
The performance of IM with different design time value

Higher time value results in higher service reliability
IM-B will be constrained by LCC budget
IM-B will be constrained by LCC budget

Upper bound from the constraint of LCC

IM-B will be constrained by **100 billion NTD LCC budget** when design time value is more than about **110,000 NTD/train-hour**
**IM-R will be constrained by design service reliability**

![Graph showing the relationship between service reliability and design time value. The graph indicates that IM-R will be constrained by design service reliability.](image-url)
**IM-R will be constrained by design service reliability**

**Lower bound from the constraint of service reliability**

IM-R will be constrained by **97.7% service reliability** when design time value is less than about 60,000 NTD/train-hour
IM-BR will be constrained by both LCC and service reliability

*IM-BR is constrained by budget and service reliability; it has the same trend as IM in terms of time value between 60,000~110,000 NTD/train-hour*
Service Reliability will be constrained by both LCC and service reliability. IM-BR will be constrained by both LCC and service reliability. This integrated model framework is flexible according to planners’ need in rail system design.
This research develops an **optimization process** to assist decision makers in optimally allocating service reliability, system reliability, and LCC.

- It is essential to incorporate **service reliability** in rail system design.

**The proposed tool can help railways maximize their return on investment and provide reliable service to passengers.**
References


Thank You!

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